



DCpic, Commutative Diagrams in a (La)TeX Document

PEDRO QUARESMA*
CISUC

DEPARTAMENTO DE MATEMÁTICA, UNIVERSIDADE DE COIMBRA
3001-454 COIMBRA, PORTUGAL

ABSTRACT. DCpic is a package of TeX macros for graphing Commutative Diagrams in a (La)TeX or ConTeXt document. Its distinguishing features are: the use of PICTEX a powerful graphical engine, and a simple specification syntax. A commutative diagram is described in terms of its objects and its arrows. The objects are textual elements and the arrows can have various straight or curved forms.

We describe the syntax and semantics of the user's commands, and present many examples of their use.

KEYWORDS: Commutative Diagrams, (La)TeX, PICTEX

INTRODUCTION

COMMUTATIVE DIAGRAMS (Diagramas Comutativos, in Portuguese), are a kind of graphs which are widely used in Category Theory [4, 7, 9], not only as a concise and convenient notation but also for “arrow chasing”, a powerful tool for mathematical thought. For example, the fact that in a Category we have arrow composition is easily expressed by the following commutative diagram.

$$\begin{array}{ccc} A & \xrightarrow{f} & B & \xrightarrow{g} & C \\ & \searrow & & \nearrow & \\ & & & & \\ & \underbrace{\hspace{10em}}_{g \circ f} & & & \end{array}$$

The word commutative means that the result from going through the path f plus g is equal to the result from going through the path $g \circ f$. Most of the graphs used

*This work was partially supported by the Portuguese Ministry of Science and Technology (MCT), under the programme PRAXIS XXI.

in Category Theory are digraphs which we can specify in terms of its objects, and its arrows.

The (La)TeX approach to typesetting can be characterized as “logical design” [5, 6, 8], but commutative diagrams are pieces of “visual design”, and that, in our opinion is the *piece de resistance* of commutative diagrams package implementation in (La)TeX. In a commutative diagrams package a user seeks the simplest notation, a logical notation, with the most powerful graphical engine possible, the visual part. The DCpic package, along with the package by John Reynolds [3, 10], has the simplest notation off all the commutative diagrams packages described in the Feruglio article [3]. In terms of graphical capabilities the PjCTeX [12] package provides us with the best TeX-graphics engine, that is, without going to *Postscript* specials.

The DCpic package depends only of PjCTeX and TeX, which means that you can use it in all formats that are based on these two. We have tested DCpic with LATEX, TeX plain, pdfLATEX, pdfTeX [11], and ConTeXt [8]; we are confident that it can be used under many other formats.

The present version (3.1) of DCpic package is available in CTAN and in the author’s Web-page¹.

CONSTRUCTING COMMUTATIVE DIAGRAMS

DCpic depends on PjCTeX, thus you must include an appropriate command to load PjCTeX and DCpic in your document, e.g. “`\usepackage{dcpic,pictex}`”, in a LATEX document.

A commutative diagram in DCpic is a “picture” in PjCTeX, in which we place our *objects* and *morphisms* (arrows). The user’s commands in DCpic are: `begin dc` and `end dc` which establish the coordinate system where the objects will be placed; `obj`, the command which defines the place and the contents of each object; `mor`, and `cmor`, the commands which define the morphisms, linear and curved arrows, and its labels.

Now we will describe each of these commands in greater detail.

The Diagram Environment

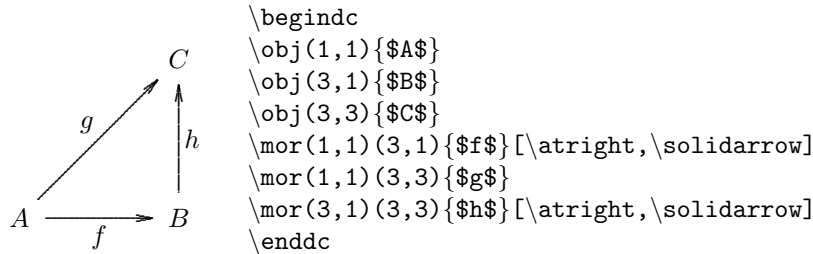
The command `begin dc`, establishes a Cartesian coordinate system with 1pt units,

```
\begin dc[magnification factor] ... \end dc
```

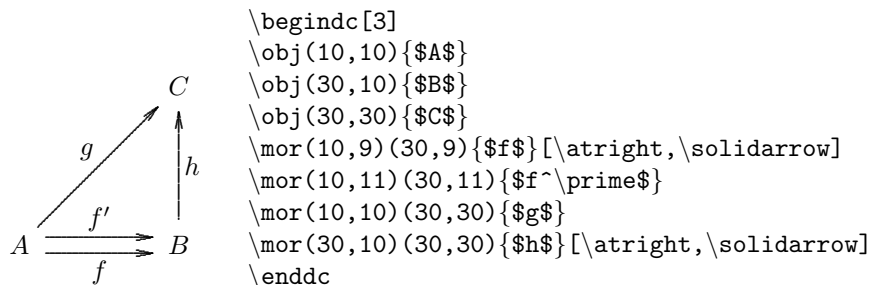
such a small unit gives us a good control over the placement of the graphical objects, but in most of the diagrams not involving curved arrows such a “fine grain” is not desirable, so the optional argument specifies a magnifying factor $m \in \mathbb{N}$, with a default value of 30. The advantage of this decision is twofold: we can define the “grain” of the diagram, and we can adjust the size of the diagram to the available space.

- ◊ a “course grain” diagram is specified almost as a table, with the numbers giving us the lines and the columns where the objects will be placed, the following diagram has the default magnification factor:

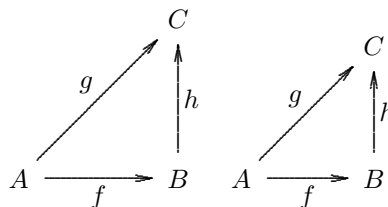
¹<http://www.mat.uc.pt/~pedro/LaTeX/>



- ◇ a “fine grain” diagram is a bit harder to design but it gives us a better control over the objects placement, the following diagram has a magnification factor of three, this gives us the capability of drawing the arrows f and f' very close together:



- ◇ the magnification factor gives us the capability of adapting the size of the diagram to the available space, without having to redesign the diagram, for example the specification of the next two diagrams differs only in the magnification factor: 30 for the first; and 25 for the second.



Note that the magnification factor does not interfere with the size of the objects, but only with the size of the diagram as a whole.

After establishing our “drawing board” we can begin placing our “objects” on it, we have three commands to do so, the `obj`, `mor`, and `cmor`, for objects, morphisms, and “curved” morphisms respectively.

Objects

Each object has a place and a content

```
\obj(<x>,<y>){<contents>}
```

the x and y , integer values, will be multiplied by the magnifying factor. The *contents* will be put in the centre of an “hbox” expanding to both sides of $(m \times x, m \times y)$.

Linear Arrows

Each linear arrow will have as mandatory arguments two pairs of coordinates, the beginning and the ending points, and a label,

```
\mor(<x1>,<y1>)(<x2>,<y2>)[<d1>,<d2>]{<label>}[<label placement>,<arrow type>]
```

the other arguments are optional. The two pairs of coordinates should coincide with the coordinates of two objects in the diagram, but no verification of this fact is made. The line connecting the two points is constructed in the following way: the beginning is given by a point 10pt away from the point $(m \times x_1, m \times y_1)$, likewise the end point is 10 points away from $(m \times x_2, m \times y_2)$. If the “arrow type” specifies that, a tail, and a pointer (arrow) will be added. The label is placed in a point (x_l, y_l) at a distance of 10 points from the middle point of the arrow, the position of the “hbox” is dependent of the angle and the direction of the arrow, if the arrow is horizontal the “hbox” will be centred in (x_l, y_l) , if the arrow is vertical the “hbox” will be left, or right, justified in (x_l, y_l) , and similarly for the other cases. In all cases the position of the “hbox” is such that the contents of it will not interfere with the line.

The distance from the point $(m \times x_1, m \times y_1)$ to the actual beginning of the arrow may be modified by the user with the specification of d_1 , the same thing happens for the arrow actual ending in which case the user-value will be d_2 . The specification of d_1 and d_2 is optional.

The placement of the label, to the left (default value), or to the right, and the type of the arrow: a solid arrow (default value), a dashed arrow, a line, an injection arrow, or an application arrow, are the last optional arguments of this command.

Quadratic Arrows

The command that draws curved lines in DCPic uses the `setquadratic` command of P₁CT₁X, this will imply a quadratic curve specified by an odd-number of points,

```
\cmor(<list of points>)_<arrow direction>(<x>,<y>){<label>}[<arrow type>]
```

the space after the list of points is mandatory. After drawing the curved line we must put the tip of the arrow on it, at present it is only possible to choose from: up, down, left, or right pointing arrow, and we must explicitly specify what type we want. The next thing to draw it is the arrow label, the placement of that label is determined by the x , and y values which give us the coordinates, after being magnified, of the centre of the “hbox” that will contain the label itself.

The arrow type is an optional argument, its default value is a solid arrow, the other possible values are a dashed arrow and a line, in this last case the arrow tip is omitted. The arrow type values are a subset of those of the `mor` command.

A rectangular curve with rounded corners is easy to specify and should cater for most needs, with this in mind we give the following tip to the user: to specify a rectangular, with rounded corners, curve we choose the points which give us the *expanded chess-horse movement*, that is, (x, y) , $(x \pm 4, y \mp 1)$, $(x \mp 1, y \pm 4)$, or $(x, y), (x \pm 1, y \mp 4)$, $(x \mp 4, y \pm 1)$, those sets of points will give us the four corners of the rectangle; to form the whole line it is only necessary to add an odd number of points joining the two (or more) corners.

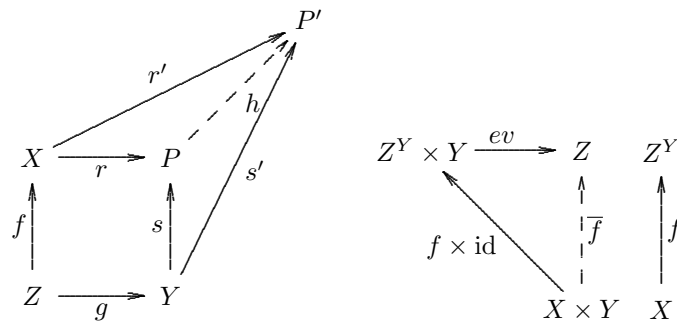
EXAMPLES

We now present some examples that give an idea of the DCpic package capabilities. We will present here the diagrams, and in the appendix the code which produced such diagrams.

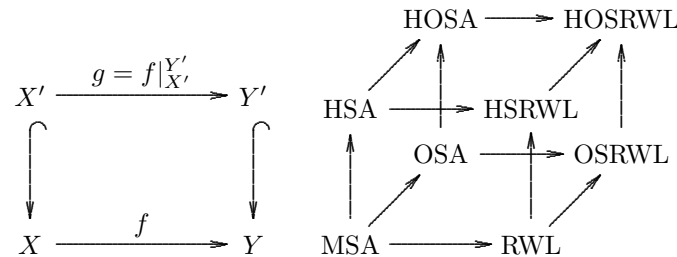
The Easy Ones

The diagrams presented in this section are very easy to specify in the DCpic syntax, just a couple of objects and the arrows joining them.

Push-out and Exponentials:



Function Restriction and the CafeOBJ Cube [2]

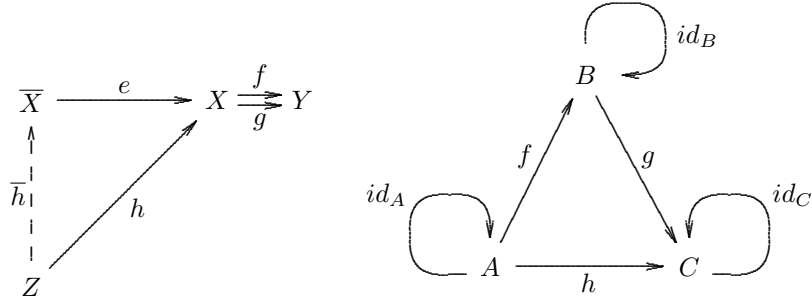


The Not so Easy

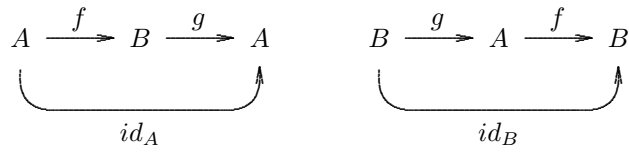
The diagrams presented in this section are a bit harder to specify. We have curved arrows, and also double arrows. The construction of the former was already explained. The double arrow (and triple, and ...) is made with two distinct arrows drawn close to each other in a diagram with a very "fine grain", that is, using a magnifying factor of just 2 or 3.

All the diagrams were made completely within DCpic.

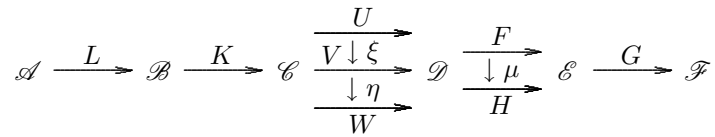
Equaliser, and a 3-Category:



Isomorphisms:



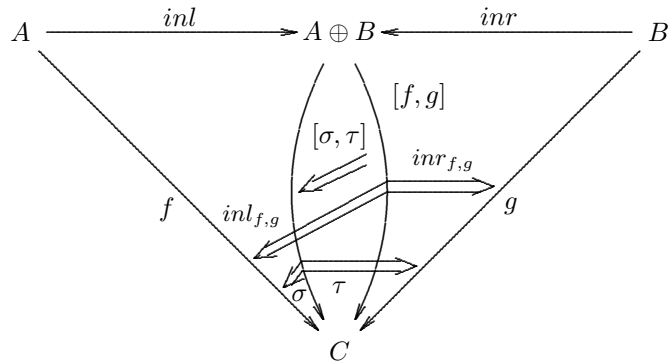
Godement's "five" rules [4]:



The others ...

It was already stated that some kinds of arrows are not supported in DCpic, e.g., \Rightarrow , but we can put a `P1CTEX` command inside a DCpic diagram, so we can produce a diagram like the one that we will show now. Its complete specification within DCpic is not possible, at least for the moment.

Lax coproduct [1]



DCPIC COMPARED

If one took the Feruglio article [3] about typesetting commutative diagrams in (La)T_EX we can say that:

- ◊ the graphical capabilities of DCpic are among the best. Excluding packages which use Postscript specials the DCpic package is the best among available packages.
- ◊ the specification syntax is one of the simplest, the package by John Reynolds has a very similar syntax.

We did not try to take any measure of computational performance.

The following diagram is one of the test-diagrams used by Feruglio, as we can see DCpic performs very well, drawing the complete diagram based on a very simple specification.

$$\begin{array}{ccccc}
 & & \Sigma^L & \xrightarrow{\varphi^r} & \Sigma^R \\
 & \nearrow \lambda^L & \downarrow i_1 & & \nearrow \lambda^R \\
 L & \xleftarrow{i_1} & L_r & \xrightarrow{r} & R \\
 \uparrow i_2 & & \uparrow i_4 & & \uparrow i_6 \\
 L_m & \xleftarrow{i_3} & K_{r,m} & \xrightarrow{r} & R_{m^*} \\
 \downarrow m & & \downarrow m & & \downarrow m^* \\
 & & \Sigma^G & \xrightarrow{\varphi^{r^*}} & \Sigma^H \\
 \downarrow \lambda^G & & \downarrow & & \downarrow \lambda^H \\
 G & \xleftarrow{i_5} & G_{r^*} & \xrightarrow{r^*} & H
 \end{array}$$

CONCLUSIONS

We think that DCpic performs well in the “commutative diagrams arena”, it is easy to use, with its commands we can produce the most usual types of commutative diagrams, and if we accept the use of P_TCT_EX commands, we are capable of producing any kind of diagram. It is also a (La)T_EX-only package, that is, the file produced by DCpic does not contain any Postscript special, neither any special font, which in terms of portability is an advantage.

The author and his colleagues in the Mathematics Department of Coimbra University have been using the (now) old version (2.1) of DCpic for some time with much success, some of the missing capabilities of the older version were incorporated in the new version (3.1), and the missing capabilities of the new version will be taken care in future versions.

REFERENCES

- [1] S. Abramsky, Dov Gabbay, and T. Maibaum, editors. *Handbook of Logic in Computer Science*, volume 1 of *Oxford Science Publications*. Clarendon Press, Oxford, 1992.
- [2] Răzvan Diaconescu and Kokichi Futatsugi. *CafeOBJ Report: The Language, Proof Techniques, and Methodologies for Object-Oriented Algebraic Specification*, volume 6 of *AMAST series in Computing*. World Scientific, 1998.
- [3] Gabriel Valiente Feruglio. Typesetting commutative diagrams. *TUGboat*, 15(4):466–484, 1994.
- [4] Horst Herrlich and George Strecker. *Category Theory*. Allyn and Bacon Inc., 1973.
- [5] Donald E. Knuth. *The TeXbook*. Addison-Wesley Publishing Company, Reading, Massachusetts, 1986.
- [6] Leslie Lamport. *LATEX: A Document Preparation System*. Addison-Wesley Publishing Company, Reading, Massachusetts, 2nd edition, 1994.
- [7] S. MacLane. *Categories for the Working Mathematician*. Springer-Verlag, New York, 1971.
- [8] Ton Otten and Hans Hagen. *ConTeXt an excursion*. Pragma ADE, Hasselt, 1999.
- [9] Benjamin Pierce. *Basic Category Theory for Computer Scientists*. Foundations of Computing. The MIT Press, London, England, 1998.
- [10] John Reynolds. *User's Manual for Diagram Macros*. <http://www.cs.cmu.edu/~jcr/>, 1987. `diagmac.doc`.
- [11] Hàn Thế Thành, Sebastian Raetz, and Hans Hagen. *The pdfTeX manual*, 1999.
- [12] Michael Wichura. *The PjCTEX Manual*. M. Pfeffer & Co., New York, 1987.

APPENDIX: THE DCPIC SPECIFICATIONS

Push-out:

```

\begin{dc}[26]
\obj(1,1){$Z$}
\obj(1,3){$X$}
\obj(3,1){$Y$}
\obj(3,3){$P$}
\obj(5,5){$P^{\prime}$}
\mor(1,1)(1,3){$f$}
\mor(1,1)(3,1){$g$}[\atright,\solidarrow]
\mor(1,3)(3,3){$r$}[\atright,\solidarrow]
\mor(3,1)(3,3){$s$}
\mor(1,3)(5,5){$r^{\prime}$}
\mor(3,1)(5,5){$s^{\prime}$}[\atright,\solidarrow]
\mor(3,3)(5,5){$h$}[\atright,\dasharrow]
\end{dc}

```


Exponentials:

```

\begin{c}
\obj(1,3){Z^Y\times Y}
\obj(3,3){ZZ}
\obj(3,1){X\times Y}
\obj(4,1){XX}
\obj(4,3){Z^Y}
\mor(1,3)(3,3)[20,10]{ev}
\mor(3,1)(1,3){f\times\mathrm{id}}
\mor(3,1)(3,3){\overline{f}}[\atright,dasharrow]
\mor(4,1)(4,3){f}[\atright,solidarrow]
\end{c}

```

Function Restriction:

```

\begin{c}[28]
\obj(1,1){X}
\obj(1,3){X^\prime}
\obj(3,1){Y}
\obj(3,3){Y^\prime}
\mor(1,1)(3,1){f}
\mor(1,3)(1,1){}[\atright,injectionarrow]
\mor(3,3)(3,1){}[\atright,injectionarrow]
\mor(1,3)(3,3){g=f|_{Y^\prime}_X^\prime}
\end{c}

```

CafeOBJ Cube:

```

\begin{c}[17]
\obj(1,1){MSA}
\obj(5,1){RWL}
\obj(3,3){OSA}
\obj(7,3){OSRWL}
\obj(1,4){HSA}
\obj(5,4){HSRWL}
\obj(3,6){HOSA}
\obj(7,6){HOSRWL}
\mor(1,1)(5,1)[15,15]{}
\mor(1,1)(1,4){}
\mor(1,1)(3,3){}
\mor(5,1)(5,4){}
\mor(5,1)(7,3){}
\mor(3,3)(3,6){}
\mor(3,3)(7,3)[15,22]{}
\mor(7,3)(7,6){}
\mor(1,4)(5,4)[15,22]{}
\mor(1,4)(3,6){}
\mor(3,6)(7,6)[17,26]{}
\mor(5,4)(7,6){}
\end{c}

```

Equaliser:

```

\begin{c}[2]
\obj(1,1){ZZ}
\obj(1,36){\overline{X}}
\obj(36,36){XX}
\obj(52,36){Y}

```

```

\mor(1,1)(1,36){$\overline{h}}[\atleft,\dasharrow]
\mor(1,1)(36,36){h}[\atright,\solidarrow]
\mor(1,36)(36,36){e}
\mor(36,37)(52,37)[8,8]{f}
\mor(36,35)(52,35)[8,8]{g}[\atright,\solidarrow]
\enddc

```

A 3-Category:

```

\begindc[3]
\obj(14,11){A}
\obj(39,11){C}
\obj(26,35){B}
\mor(14,11)(39,11){h}[\atright,\solidarrow]
\mor(14,11)(26,35){f}
\mor(26,35)(39,11){g}
\cmor((11,10)(10,10)(9,10)(5,11)(4,15)(5,19)(9,20)(13,19)(14,15))
\pdown(1,20){id_A}
\cmor((42,10)(43,10)(44,10)(48,11)(49,15)(48,19)(44,20)(40,19)(39,15))
\pdown(52,20){id_C}
\cmor((26,39)(27,43)(31,44)(35,43)(36,39)(35,36)(31,35)) \pleft(40,40){id_B}
\enddc

```

Isomorphisms:

```

\begindc[3]
\obj(10,15){A}
\obj(40,15){A}
\obj(25,15){B}
\mor(10,15)(25,15){f}
\mor(25,15)(40,15){g}
\cmor((10,11)(11,7)(15,6)(25,6)(35,6)(39,7)(40,11)) \pup(25,3){id_A}
\obj(55,15){B}
\obj(85,15){B}
\obj(70,15){A}
\mor(55,15)(70,15){g}
\mor(70,15)(85,15){f}
\cmor((55,11)(56,7)(60,6)(70,6)(80,6)(84,7)(85,11)) \pup(70,3){id_B}
\enddc

```

Godement's "five" rules:

```

\begindc[7]
\obj(12,10){$\mathcal{A}}
\obj(19,10){$\mathcal{B}}
\obj(26,10){$\mathcal{C}}
\obj(34,10){$\mathcal{D}}
\obj(41,10){$\mathcal{E}}
\obj(48,10){$\mathcal{F}}
\mor(12,10)(19,10){L}
\mor(19,10)(26,10){K}
\mor(26,10)(34,10){V\quad}
\mor(26,12)(34,12){U}
\mor(26,12)(34,12){$\downarrow\xi}[\atright,\solidarrow]
\mor(26,8)(34,8){$\downarrow\eta}
\mor(26,8)(34,8){W}[\atright,\solidarrow]
\mor(34,11)(41,11){F}
\mor(34,9)(41,9){$\downarrow\mu}
\mor(34,9)(41,9){H}[\atright,\solidarrow]

```

```
\mor(41,10)(48,10){$G$}
\enddc
```

Lax coproduct: Guess how.

DCpic and the others:

```
\begin{dc}[35]
\obj(1,1){$G$}
\obj(3,1){$G_{r^*}$}
\obj(5,1){$H$}
\obj(2,2){$\Sigma^G$}
\obj(6,2){$\Sigma^H$}
\obj(1,3){$L_m$}
\obj(3,3){$K_{r,m}$}
\obj(5,3){$R_{m^*}$}
\obj(1,5){$L$}
\obj(3,5){$L_r$}
\obj(5,5){$R$}
\obj(2,6){$\Sigma^L$}
\obj(6,6){$\Sigma^R$}
\mor(1,1)(2,2){$\lambda^G$}
\mor(3,1)(1,1){$i_5$}[\atleft,\aplicationarrow]
\mor(3,1)(5,1){$r^*$}[\atright,\solidarrow]
\mor(5,1)(6,2){$\lambda^H$}[\atright,\dasharrow]
\mor(2,2)(6,2){$\varphi_{r^*}$}[\atright,\solidarrow]
\mor(1,3)(1,1){$m$}[\atright,\solidarrow]
\mor(1,3)(1,5){$i_2$}[\atleft,\aplicationarrow]
\mor(3,3)(1,3)[14,10]{$i_3\text{\quad}$}[\atright,\aplicationarrow]
\mor(3,3)(5,3)[14,10]{$r$}
\mor(3,3)(3,5){$i_4$}[\atright,\aplicationarrow]
\mor(3,3)(3,1){$\stackrel{\displaystyle m}{\bar{B}}$}
\mor(5,3)(5,5){$i_6$}[\atright,\aplicationarrow]
\mor(5,3)(5,1){$\stackrel{\displaystyle m^*}{\bar{A}}$}
\mor(1,5)(2,6){$\lambda^L$}
\mor(3,5)(1,5){$i_1\text{\quad}$}[\atright,\aplicationarrow]
\mor(3,5)(5,5){$r$}
\mor(5,5)(6,6){$\lambda^R$}[\atright,\solidarrow]
\mor(2,6)(2,2){$\varphi_m$}[\atright,\solidarrow]
\mor(2,6)(6,6){$\varphi_r$}
\mor(6,6)(6,2){$\varphi_{m^*}$}
\end{dc}
```